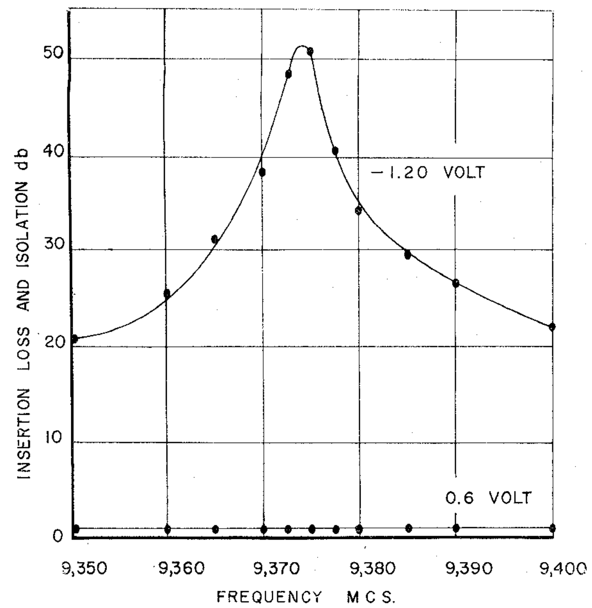

Fig. 11—Frequency dependence of hybrid-tee switch characteristic of both silicon and germanium diodes.

switch has proved to be faster than the ferrite switch. As far as the solid-state material is concerned, both germanium and ferrite have relaxation times less than 1 mμsec. It is considerably more difficult to develop fast rise time magnetic pulses (20 to 30 oersteds) for ferrite switching than it is to develop fast rise time voltage pulses for semiconductor switching. Ferrite switches are indicated for high-power high-speed microwave switching and semiconductor switches are indicated for low-power high-speed microwave switching.[8]

Acknowledgment

The authors thank R. D. Hatcher, R. C. LeCraw, R. F. Sullivan and H. B. Bruns for many helpful discussions and suggestions.

[8] Since the writing of this article, another microwave diode switch has been proposed by A. Uhlir, Jr., "The potential of semiconductor diodes in high-frequency communications," Proc. IRE, vol. 46, pp. 1099–1115; June, 1958. If his theory results in another practical switch, the techniques reported here should be equally useful with either switch.

Microwave Q Measurements in the Presence of Coupling Losses*

E. L. GINZTON†

Summary—In the use of the impedance (Q-circle) method of measuring the cavity Q values, the presence of losses in the coupling network (between the cavity and the available external terminals) is usually neglected. If appreciable losses are present this simplification is not justified, and its use can lead to significant errors.

The losses in any coupling network can be described by means of an equivalent canonical circuit containing a series and a shunt resistor. The losses due to the series element are immediately apparent from the character of the impedance locus when plotted on a Smith Chart and can be corrected for an "apparent" Q value. However, unless the shunt loss can be determined by a separate calibration of the coupling network, the apparent Q value will be ambiguous because the shunt losses occurring in the coupling network are not distinguishable from those in the cavity proper.

Methods for using the impedance data for determining the Q values are given on the assumption that the coupling network parameters can be found. It is also pointed out that due to the presence of coupling losses the loaded and external Q values are no longer uniquely defined, but their meaning depends upon the application of interest. Formulas relating these to the coupling network parameters are given.

* Manuscript received by the PGMTT, February 12, 1958; revised manuscript received, May 26, 1958.
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Introduction

A COMMON useful method of measuring the Q values of a microwave cavity consists of measuring the self-impedance of the cavity as a function of frequency. The equivalent circuit of the main elements of apparatus needed for this measurement is shown schematically in Fig. 1, where the cavity is shown as if it were a lumped-constant resonant circuit inductively coupled to the uniform transmission line (which contains a slotted section for impedance measurements). This special form of the equivalent circuit has been shown to be sufficiently general and accurate for most practical cases: the resonance phenomenon occurs within the cavity so that the losses within it can be represented by the resistor in series with L_2 and C_2.

The losses in the *coupling network*, *i.e.*, in the elements which transfer energy from the transmission line into the cavity, are generally very small and usually their presence can be neglected. The theory of the experiment required to determine the Q values, details of

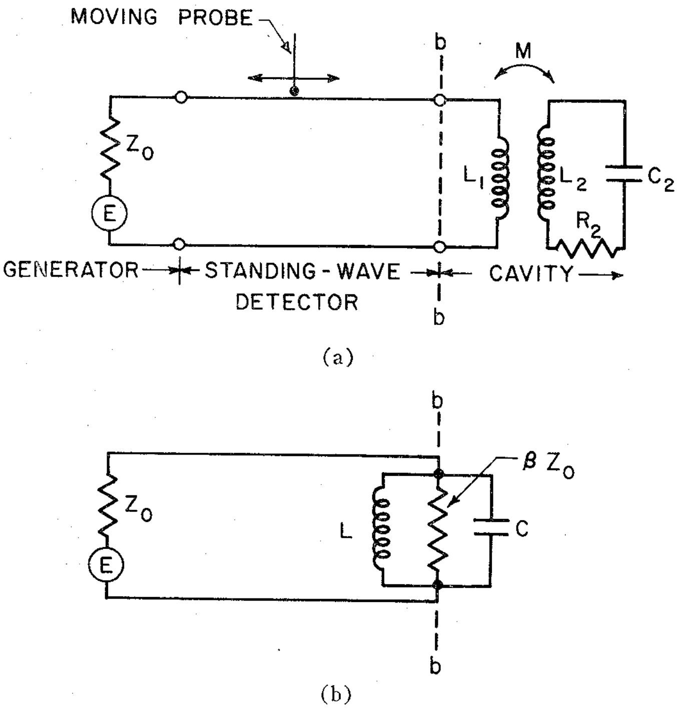

Fig. 1—(a) Schematic representation of the apparatus used in impedance method of cavity-Q measurement. (b) The equivalent circuit at the detuned short position for the negligible coupling-loss case.

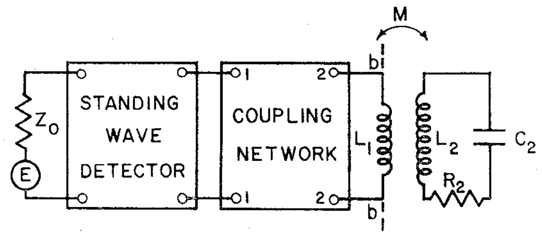

Fig. 2—The general form of the equivalent circuit representing the apparatus shown in Fig. 1(a).

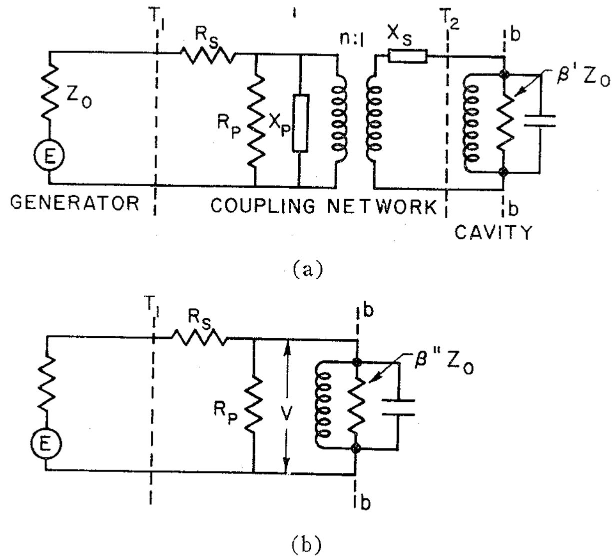

Fig. 3—(a) Representation of the coupling network in the canonical form. (b) Simplification of (a) transforming the elements on the secondary side to the primary.

measurement, and interpretation of the impedance data, sources of error, helpful techniques, etc., are discussed in numerous sources.[1,2]

The assumption that the coupling network is *lossless* is not always accurate. If sufficient losses are present, the available methods for determination of Q are not valid in general and their use can lead to significant errors. This fact has been recognized previously, and a special case in which the coupling loss can be represented by a simple resistance in series with the coupling element has been described.[2–4]

The purpose of this paper is to discuss the more general case in which the losses in the coupling network need not be presumed to be of the series type but can be of the series, shunt, or distributed form.

The Equivalent Circuit

The general form of the equivalent circuit representing the cavity and its coupling network is shown in Fig. 2. The terminals of the *coupling network* 1-1 and 2-2 are presumed to be selected so that all sources of loss within the network are included between these terminals. Further, the terminals 1-1 are to be located within the uniform transmission line of the standing-wave detector and are to be selected in a manner which will simplify the interpretation and manipulation of the laboratory data. Specifically, terminals 1-1 will be located at the position which is called the *detuned-short* (DS) position, in analogy with the unambiguous location of such terminals in the absence of losses in the coupling network; the actual location of these terminals will be discussed below.

There are several forms of an equivalent circuit which can be used to describe the coupling network: the L, T, π, lattice, etc. In this case, it is convenient to use the canonical network shown in Fig. 3(a).[5–7] In this network, the resistances R_s and R_p represent the dissipative losses of the network; the reactances X_p and X_s, together with the location of the input reference plane T_1, represent the reactive elements. The equivalent circuit of Fig. 3(a) can be simplified as shown in Fig. 3(b), where the new resonant circuit has a resonant impedance $\beta'' Z_0$ after transformation through the transformer. Due to the presence of various coupling reactances, which are assumed to be small, the resonant

[1] For example, E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 9; 1957.

[2] A. Singh, "An improved method for the determination of Q of cavity resonators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 155–160; April, 1958.

[3] L. Malter and G. R. Brewer, "Microwave Q measurements in the presence of series losses," *J. Appl. Phys.*, vol. 20, pp. 918–925; October, 1949.

[4] Ginzton, *op. cit.*, pp. 424–428.

[5] A. Weissfloch, "Circle geometric four-terminal network theory; its significance as a circuit theory at microwaves," *Hochfrequenz. Elektr.*, pp. 100–123; April, 1943.

[6] L. B. Felsen and A. A. Oliner, "Determination of equivalent circuit parameters for dissipative microwave structures," Proc. IRE, vol. 42, pp. 477–483; February, 1954.

[7] Ginzton, *op. cit.*, pp. 323–326, 328–329.

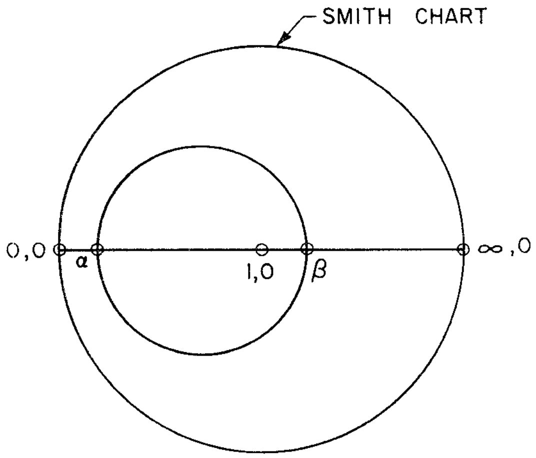

Fig. 4—Variation of the cavity impedance with frequency in the presence of coupling loss (plotted on a Smith Chart).

frequency of the circuit shown in Fig. 3(b) is no longer equal to the resonant frequency of the cavity itself; however, this is of no particular importance, just as the presence of the small coupling reactances in the case of negligible losses is immaterial.

The variation of impedance with frequency of the circuit shown in Fig. 3(b) at terminals 1-1 is indicated in Fig. 4 for a typical case. This locus must be a circle, since the transformation of the circular impedance locus of the impedance at terminals b-b in Fig. 3 through the coupling network is conformal.

For simplicity, the terminals 1-1, henceforth called the reference plane T_1 or merely T_1, are chosen such that the impedance locus in Fig. 4 is symmetrical about the resistive axis. (Actually, there are two such planes $\lambda_t/4$ apart; the one that makes $\alpha<1$ is selected, as indicated in Fig. 4.) If the series resistance in Fig. 3(b) were zero, the impedance locus would pass through the origin ($R=0$, $X=0$); hence, the location of T_1 in the manner stated corresponds to the location of the "detuned-short position" in the no-loss case.

Referring to Fig. 4, let the intercepts of the impedance locus with the normalized resistive axis of the Smith Chart be called α and β which, by comparison with Fig. 3(b), are:

$$\alpha = \frac{R_s}{Z_0} \tag{1}$$

$$\beta = \alpha + \beta' \tag{2a}$$

where

$$\beta' = \frac{R_p\beta''}{R_p + \beta'' Z_0},$$

or

$$\beta' = \beta'' \frac{\gamma}{\gamma + 1} \tag{2b}$$

where

$$\gamma = R_p/\beta'' Z_0.$$

The intercepts described by α and β result, respectively, when the cavity is tuned far off resonance and when it is tuned to the angular frequency ω_0 (at which the impedance across R_p becomes a maximum and purely resistive). For descriptive purposes, quite arbitrarily, the angular frequency ω_0 may be called the *resonant frequency*.

Referring to Fig. 3(b), the impedance Z_{11} at T_1 can be represented by

$$Z_{11} = R_s + \frac{R_p Z_{bb}}{R_p + Z_{bb}}, \tag{3}$$

but

$$\frac{Z_{bb}}{Z_0} = \frac{\beta''}{1 + j2Q_0\delta} \tag{4}$$

where Z_0 = characteristic impedance of the input transmission line. Q_0 is the unloaded Q value of the resonant cavity, and

$\delta = (\omega - \omega_0)/\omega$ = frequency tuning parameter
ω = angular frequency $= 2\pi f$
ω_0 = angular frequency at which the input impedance is maximum and real.

Combining (3) and (4) and rearranging,

$$\frac{Z_{11}}{Z_0} = \alpha + \frac{\beta - \alpha}{1 + j2Q_0'\delta}, \tag{5}$$

where

$$Q_0' = Q_0 \left\{ \frac{1}{1 + \frac{\beta'' Z_0}{R_p}} \right\} \tag{6a}$$

$$= Q_0 \frac{\gamma}{\gamma + 1}. \tag{6b}$$

Thus, the Smith Chart impedance locus as seen at T_1 is modified by the presence of loss in the two ways indicated in Fig. 5. The dashed circle shows the locus that would be obtained if there were no coupling losses. If the series losses alone were present ($R_p = \infty$), the impedance locus would pass through the point (α, 0) instead of (0, 0). If the shunt losses alone were present ($R_s = 0$), the circle would pass through (0, 0) but would correspond to a cavity with a lowered Q_0. The solid circle shown is one that would be obtained if both losses were present simultaneously.

General Discussion and Determination of Q_0

It is apparent from the above discussion that it is necessary to know the coupling network parameters R_s and R_p to permit the interpretation of the impedance information available at terminals 1-1. The constants R_s and R_p could be found experimentally if the coupling network at its output terminals could be replaced by a moving short circuit to permit separate experimental determination of the parameters of the canoni-

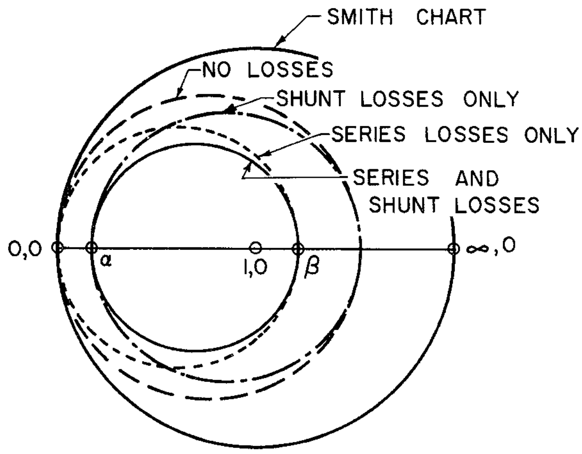

Fig. 5—The effect of coupling network losses upon the impedance locus.

cal circuit.[8] If these parameters were experimentally determined, there would be two possible approaches to the determination of cavity Q: the measured values of impedance Z_{11} could be converted to the values of impedance Z_{bb} at the cavity terminals, and the usual well-known procedures to analyze the resultant *ideal* impedance locus could be used; or, the measured impedance locus could be analyzed directly using the approach which is described below.

In many cases, however, the separation of the cavity from its coupling network is impractical or impossible; this leads to difficulties which are discussed more fully further on. It should be obvious that the value of parameter R_s (*i.e.*, α) can be determined immediately from the experimentally determined impedance locus at terminals 1-1.

Consider for the moment that the parameters α, β, and γ are all known. Since in many practical cases the coupling network losses are small and are due to *either* series or shunt elements, it is convenient at first to consider these two cases separately from the more general case.

Case 1) Shunt Losses Only

In this case, $\alpha=0$, and (5) becomes

$$Z_{11} = \frac{\beta}{1 + j2Q_0'\delta} \cdot \qquad (7)$$

The apparent value Q_0' can be found using the measured impedance locus.[9] The true value of Q_0 can be found from (6) which is repeated below.

$$Q_0 = Q_0' \frac{\gamma + 1}{\gamma} \cdot \qquad (8)$$

The evaluation of the true value of Q_0 *requires* the knowledge of the parameter γ.

[8] The parameters of the canonical network can be found as described in Felsen and Oliner, *op. cit.*, or Ginzton, *op. cit.*, pp. 343–345.
[9] See, for instance, Ginzton, *op. cit.*, pp. 406–424.

Case 2) Series Losses Only

In this case, $R_p = \infty$, and (5) becomes:

$$\frac{Z_{11}}{Z_0} = \alpha + \frac{\beta - \alpha}{1 + j2Q_0\delta} \cdot \qquad (9)$$

Assuming that an experimental impedance locus such as is shown in Fig. 4 has been obtained by measuring the impedance at a number of frequencies which are also measured, the Q_0 value can be found as follows.

Let $2Q_0\delta = \pm 1$. Substituting this into (9) and simplifying:

$$\frac{Z_{11}}{Z_0} = \frac{\beta + \alpha \pm j(\beta - \alpha)}{2} \cdot \qquad (10)$$

Using the values of α and β obtained from the experimental plot for the given locus, the real and imaginary parts of (10) can be computed and located on the locus. This determines the two points on the locus for which $2Q_0\delta = \pm 1$. If the frequencies at which they occur are found to be f_1 and f_2, then

$$Q_0 = \delta_1 - \delta_2 = \frac{f_1 - f_2}{f_0} \cdot \qquad (11)$$

If the two points on the locus so determined do not correspond to the points for which the frequencies have been measured, the desired frequencies f_1 and f_2 can be found by means of an *auxiliary linear frequency scale* which can be constructed as indicated in Fig. 6, the geometrical construction for which is justified in the Appendix. Let points a, b, c, d, e, and f represent a set of impedances measured at frequencies f_a, f_b, f_c, f_d, f_e, and f_f, respectively. A line $\overline{AB}$ is drawn perpendicularly to the resistive axis and radial lines are drawn from $(\alpha, 0)$ through the known impedance points. The intercepts along line AB are labeled with the known frequency $f_a \cdots f_e$ and are linear in frequency, thus permitting the determination of the frequency of any point along the impedance locus.

Case 3) Series and Shunt Losses

If both the series and shunt losses are present, and each is small, the Q_0' value can be found, first using (11) which takes into account the presence of the series losses alone; the effect of the presence of the shunt losses can then be taken into account by multiplying this value by the correction factor given by (8).

If the losses are not small, the known parameters of the coupling network permit the calculation of the impedance at the reference plane T_1 for the half-power points (*i.e.*, $2Q_0\delta = \pm 1$). The two frequencies at which these impedances are actually found define the half-power bandwidth from which the Q_0 value can be computed. Alternatively, the impedance at T_1 can be measured as a function of frequency and each of the measured values converted by computation, resulting in the ideal impedance locus which can be analyzed by conventional means.

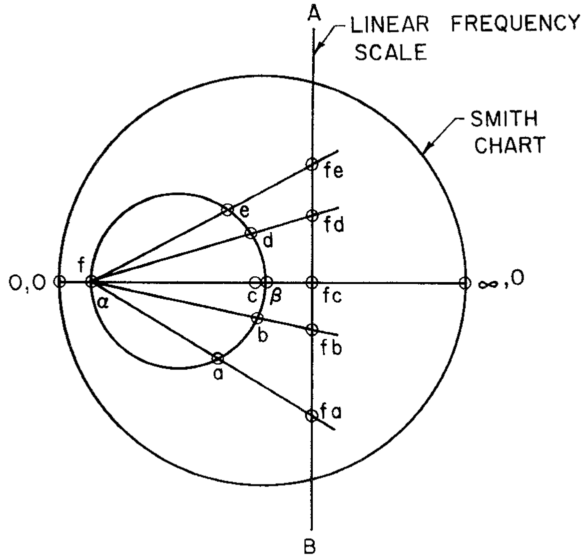

Fig. 6—Geometrical construction used to establish a linear frequency scale for interpolation between measured impedance points.

In some cases, the coupling network may happen to be an integral part of the cavity, making it impossible or impractical to determine the network constants. In this case, the observation of the impedance locus at the input of the coupling network allows the evaluation of the series losses α and the apparent value Q_0'. The separation of losses occurring within the resonant cavity and the coupling network becomes *impossible;* if this distinction happens to be important, some expedient must be found to separate the losses in the two parts of the structure. Sometimes this can be done by constructing an auxiliary structure susceptible to physical dismounting to permit a more complete experimental study of its parts.

Determination of Loaded and External Q Values

In addition to the natural or unloaded Q values of the cavity, Q_0, other Q values are important in the usual practice. These normally are Q_L and Q_{ext}, the loaded and external Q values, respectively. In the presence of losses, still other Q values become relevant, as defined below. Using the conventional definition for Q, the following descriptive statements can be made:

$$Q_0 = 2\pi f \frac{\text{energy stored}}{\text{energy lost in the cavity proper}} \tag{12}$$

$$Q_0' = 2\pi f \frac{\text{energy stored}}{\text{energy lost in cavity and shunt element of coupling network}} \tag{13}$$

$$Q_0'' = 2\pi f \frac{\text{energy stored}}{\text{energy lost in cavity and coupling network}^9} \tag{14}$$

$$Q_L = 2\pi f \frac{\text{energy stored}}{\text{energy lost in cavity, coupling network, and load}} \tag{15}$$

$$Q_{ext} = 2\pi f \frac{\text{energy stored}}{\text{energy lost in the load}}\cdot \tag{16}$$

For reference, in the absence of coupling network losses, these can be written as follows, using the notation shown in Fig. 1(b) (with the cavity coupled through the coupling network to a source with impedance equal to Z_0).[10]

$$Q_0 = \frac{\beta Z_0}{\omega L} = \omega C \beta Z_0 \tag{17}$$

$$Q_L = \omega C \frac{\beta Z_0^2}{\beta Z_0 + Z_0} \tag{18}$$

$$Q_{ext} = \omega C Z_0 \tag{19}$$

$$Q_0 = Q_0' = Q_0'' \tag{20}$$

and

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}\cdot \tag{21}$$

In the presence of coupling losses, the meaning of Q_0, Q_0', Q_0'', and Q_L remains clear and unambiguous. In the case of Q_{ext}, it is necessary to be specific about the meaning of the *load*. Referring to Fig. 3(b), it is natural to regard the source impedance Z_0 as the "load" upon the network and the cavity. However, in the design of certain electron devices, it is convenient to include part or all of the loss in the coupling network with the load. This leads to the following three cases.

Case 1)

Consider Z_0 alone as the load. In this case, the expression corresponding to (21) becomes:

$$\frac{1}{Q_L} = \frac{1}{Q_0''} + \frac{1}{Q_{ext}}\cdot \tag{22}$$

Case 2)

Consider Z_0 and R_s as the load, and

$$\frac{1}{Q_L} = \frac{1}{Q_0'} + \frac{1}{Q_{ext}}\cdot \tag{23}$$

Case 3)

Consider Z_0, R_s, and R_p as the load and

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}\cdot \tag{24}$$

Assuming that the values of Q_0' and/or Q_0 have been experimentally obtained as described above, and assuming that the parameters α, β, and γ are known, the expressions for Q_L and Q_{ext} for the three cases can be written as follows.

Referring to Fig. 3(b), if V is the voltage across C, then, from (12),

[10] *Ibid.*, pp. 393–397.

$$Q_0 = 2\pi f \frac{\frac{1}{2} CV^2}{\frac{1}{2}\left(\frac{V^2}{\beta'' Z_0}\right)} \tag{25}$$

$$Q_0 = \omega C \beta'' Z_0. \tag{26}$$

Similarly,

$$Q_0' = \omega C \beta'' Z_0 \frac{1}{1 + \beta'' Z_0 / R_p} \tag{27}$$

$$= \frac{\gamma}{\gamma + 1} \tag{28}$$

and,

$$Q_0'' = Q_0' \frac{(1+\alpha)^2}{1 + \alpha(2+\beta)} \tag{29}$$

$$= Q_0 \frac{\gamma}{1+\gamma} \cdot \frac{(1+\alpha)^2}{1+\alpha(2+\beta)} \tag{30}$$

$$Q_L = Q_0' \left(\frac{1+\alpha}{1+\beta}\right) \tag{31}$$

$$= Q_0 \frac{\gamma}{1+\gamma} \cdot \frac{1+\alpha}{1+\beta} \cdot \tag{32}$$

The Q external values can be found from (16) for the three conditions stated above or by combining (22), (23), and (24), with (29), (28), and (26), respectively. These lead to:

Case 1)

$$Q_{ext} = Q_0' \frac{(\alpha + 1)^2}{\beta - \alpha} \tag{33}$$

Case 2)

$$Q_{ext} = Q_0' \frac{1 + \alpha}{\beta - \alpha} \tag{34}$$

Case 3)

$$Q_{ext} = Q_0 \frac{1+\alpha}{\beta - \alpha + (\beta+1)/\gamma} \cdot \tag{35}$$

Thus, if α, β, and γ are all known, it is possible to determine the various Q values by using the appropriate relations. If γ is not known, the remaining Q values can still be computed, but only in terms of the *apparent* Q value, Q_0', a value which does not distinguish between the losses in the cavity and the shunt element of the coupling network.

Conclusion

It has been shown that an arbitrary coupling network between a uniform transmission line and a cavity can be represented by means of an equivalent network consisting of series and shunt resistors which, qualitatively, represent the series and the shunt losses in the coupling network. If both of these can be measured by an independent calibration experiment, the Q_0 of the cavity can be determined unambiguously from the measurements of input impedance in the uniform transmission line as a function of frequency. If separate measurements cannot be made, the measurement of input impedance as a function of frequency permits the determination only of the apparent Q value of the cavity, Q_0'. Unless the shunt losses in the coupling network are separately evaluated, this apparent Q value does not distinguish between the losses in the coupling network and the cavity proper.

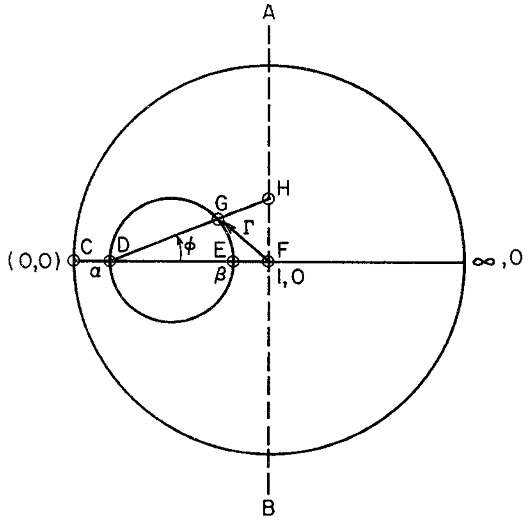

Fig. 7—Geometrical construction used to prove the linearity of the frequency scale.

Appendix

Construction of a Linear Frequency Scale

Fig. 6 shows the geometrical construction which may be used to establish a linear frequency scale in order to obtain frequencies for points along an impedance locus for which experimental frequency values are not available.

Referring to Fig. 7, draw a straight line perpendicular to the resistive axis; although the location of this line is immaterial, for the sake of simplicity in the proof, assume that it passes through the point ($R=1$, $X=0$). Radial lines originating from the point (α, 0) and passing through the point on the impedance locus for which the frequencies are known provide intercepts along the lines A-B in frequency units. It will be shown below that the frequency along the axis A-B is linear so that the frequency of any point on the locus can be determined by projecting a radial line from (α, 0) to A-B and reading the distance along A-B in frequency units (as established from the known frequency points). The proof of the construction is as follows.

Using the large circle in Fig. 7 to represent the *reflection coefficient* plane, the vector $\overline{FG}$ is the reflection coefficient Γ. The impedance value along the impedance locus is given by (5). The corresponding values of the reflection coefficient can be computed from

$$\Gamma = \frac{Z_{11} - 1}{Z_{11} + 1} \tag{36}$$

or, combining with (5),

$$\Gamma = \frac{(\beta - 1) + j2Q_0\delta(\alpha - 1)}{(\beta + 1) + j2Q_0\delta(\alpha + 1)} \cdot \tag{37}$$

Thus, when $2Q_0\delta = \pm\infty$,

$$\overline{DF} = |\Gamma_\infty|$$

$$\overline{DF} = \frac{1 - \alpha}{1 + \alpha} \cdot \tag{38}$$

The distance

$$\overline{DG} = \overline{DF} + \Gamma$$

$$\overline{DG} = \frac{1 - \alpha}{1 + \alpha} + \frac{(\beta - 1) + j2Q_0\delta(\alpha - 1)}{(\beta + 1) + j2Q_0\delta(\alpha + 1)} \tag{39}$$

$$= \frac{2(\beta - \alpha)}{(\alpha + 1)(\beta + 1) + j(\alpha + 1)^2 2Q_0\delta} \cdot \tag{40}$$

The phase angle Φ, shown in Fig. 7, is equal to argument $\overline{DG}$ and is found from the ratio of imaginary to real parts of (40):

$$\Phi = \tan^{-1} \frac{2Q_0\delta(\alpha + 1)}{\beta + 1} \cdot \tag{41}$$

The distance $\overline{FH}$, being proportional to $\tan \Phi$, is proportional to δ, *i.e.*, the frequency. Thus, the points projected from the locus onto the straight line *A-B* produce intercepts whose lengths are proportional to frequency (*Q.E.D.*).

The Excitation of a Dielectric Rod by a Cylindrical Waveguide*

C. M. ANGULO† AND W. S. C. CHANG‡

Summary—**This paper is a theoretical analysis of the excitation of the lowest circular symmetric TM surface wave along an infinite circular dielectric rod by a metallic cylindrical waveguide coaxial with the rod. The asymptotic expressions for all the fields are obtained by means of the Wiener-Hopf method. The expressions for the total average power transmitted to the surface wave, the total average power reflected, and the total power radiated, per unit incident power, are derived and computed for $\epsilon = 2.49$ for various radii of the dielectric rod.**

Introduction

It is well known that a TM circular symmetric surface wave can be easily launched along a circular dielectric rod by a metallic cylindrical waveguide. A condensed theoretical analysis of an idealized version of this problem is given here. For the detailed analysis, the reader is referred to a previous report by the authors.[1]

The structure under consideration is represented in Fig. 1. It consists of an infinite circular dielectric rod of relative permittivity ϵ and radius a fitted tight into a semi-infinite cylindrical waveguide of infinitely thin metallic wall which extends from $z = -\infty$ to $z = 0$.

The incident energy is carried by the $TM_{0,1}$ mode of the cylindrical metallic waveguide. It excites a TM surface wave along the rod, a reflected wave in the waveguide, and a scattered radiation at the end of the metallic waveguide. It is assumed here that along the dielectric rod only the lowest circular symmetric surface wave can exist and that the $TM_{0,1}$ mode is the only mode propagating inside the waveguide. This is true if $2.405(\epsilon - 1)^{-1/2} < Ka < 5.52\epsilon^{-1/2}$, where $K = 2\pi/\lambda_0$.

Since the structure considered (see Fig. 1) is independent of ϕ and the incident wave is the $TM_{0,1}$ mode, only the circular symmetric TM proper and improper modes are excited. Therefore, $\partial/\partial\phi = 0$ and $H_\rho = E_\phi = H_z = 0$. Furthermore, all the higher TM modes excited inside the cylindrical guide attenuate exponentially in the negative z direction. It follows immediately that the far zone fields of our problem must be of the forms:

$$E_z = AJ_0(K_c\rho) \exp[-j(K^2\epsilon - K_c^2)^{1/2}z] + BJ_0(K_c\rho) \exp[j(K^2\epsilon - K_c^2)^{1/2}z] \tag{1}$$

$$H_\phi = \frac{jA\omega\epsilon\epsilon_0}{K_c} J_1(K_c\rho) \exp[-j(K^2\epsilon - K_c^2)^{1/2}z] + j\frac{B\omega\epsilon\epsilon_0}{K_c} J_1(K_c\rho) \exp[j(K^2\epsilon - K_c^2)^{1/2}z] \tag{2}$$

* Manuscript received by the PGMTT, February 14, 1958; revised manuscript received, June 2, 1958. The research described in this paper has been sponsored by the Cambridge AF Res. Ctr. under Contract AF 19(604)-1391 with Brown University.

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‡ Electronics Res. Lab., Stanford University, Stanford, Calif. Formerly at Brown University, Providence, R. I.

[1] C. M. Angulo and W. S. Chang, "Excitation of a Dielectric Rod by a Cylindrical Waveguide," Div. of Eng., Brown University, Providence, R. I., Scientific Rep. AF 1391/7; July, 1957.